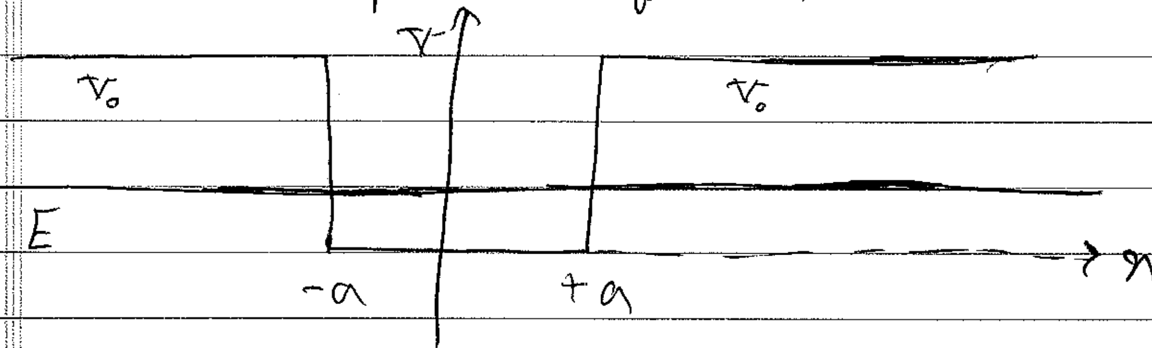
Lec 14: 09/25/2009

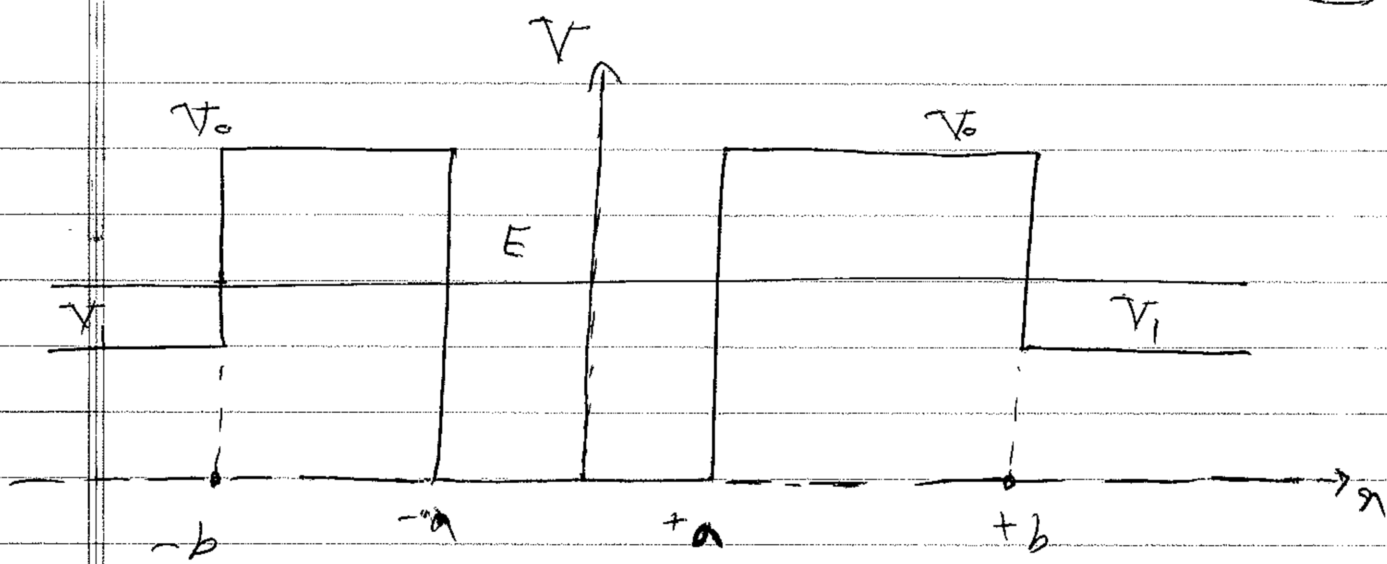
Tunneling:

Consider the square well potential:



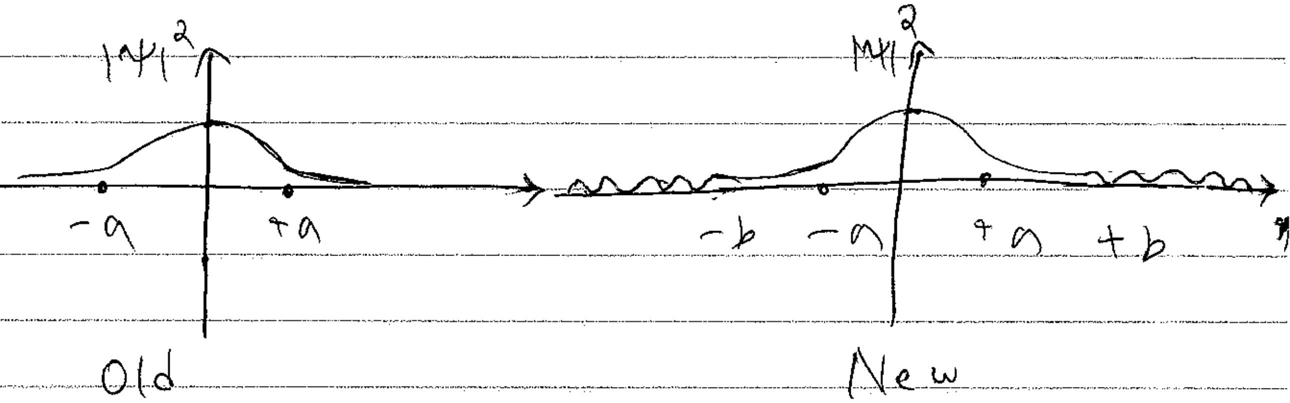
A state with energy $E < V_0$ (there is alway at least one such state) is a bound state and stable, i.e. $|\psi_{(x,t)}|^2$ is independent of time. Note that ψ is an exponentially damped function at $x > +a$, $x < -a$.

Now, lets assume that the potential is lowered from V_0 to $V_1 < V_0$ for $x > +b$, $x < -b$ where $|b| > |a|$. The question is what will happen to the state ψ that was a stable eigenstate of the Hamiltonian.



The Hamiltonian has an eigenstate with energy E in this case too. In fact, the energy spectrum is continuous for $E > V_1$.

However, ψ cannot be the eigenstate with energy E now. For the new potential, the energy eigenstate is oscillatory at $x > +b$, $x < -b$, while for the old potential the eigenstate is exponentially damped.



Therefore, the eigenstate of the old potential will be a superposition of the eigenstates of the new potential. It will be time-dependent and not stable any more. It is a metastable state that leaks outside the $[-a, +a]$ interval.

This is totall against classical intuition. In classical mechanics, if $E < V_1$, the particle cannot climb over the barrier and will always remain within the $[-a, +a]$ interval.

Why doesn't this work in quantum mechanics?

It all goes down to nonzero commutator of X and H. In a given energy eigenstate, $\Delta X \neq 0$ because the energy eigenstate is not an X eigenstate. Hence there is an uncertainty in the position and the particle has a nonzero probability to to be beyond the well.

On the other hand, if the particle is in an eigenstate of X, it will not have definite energy. In this case $\Delta H \neq 0$, and the components with $E > V_1$ can take the particle over the barrier.

Therefore a particle localized inside the $[-a, +a]$ interval, is in an unstable state. If one waits for a sufficiently long time, there is a probability to find the particle at $x > +b$ or $x < -b$. This is called tunneling, which is a purely quantum mechanical effect.

Lets consider two important examples of tunneling.

Nuclear α Decay:

α particle consists of two protons and two neutrons.

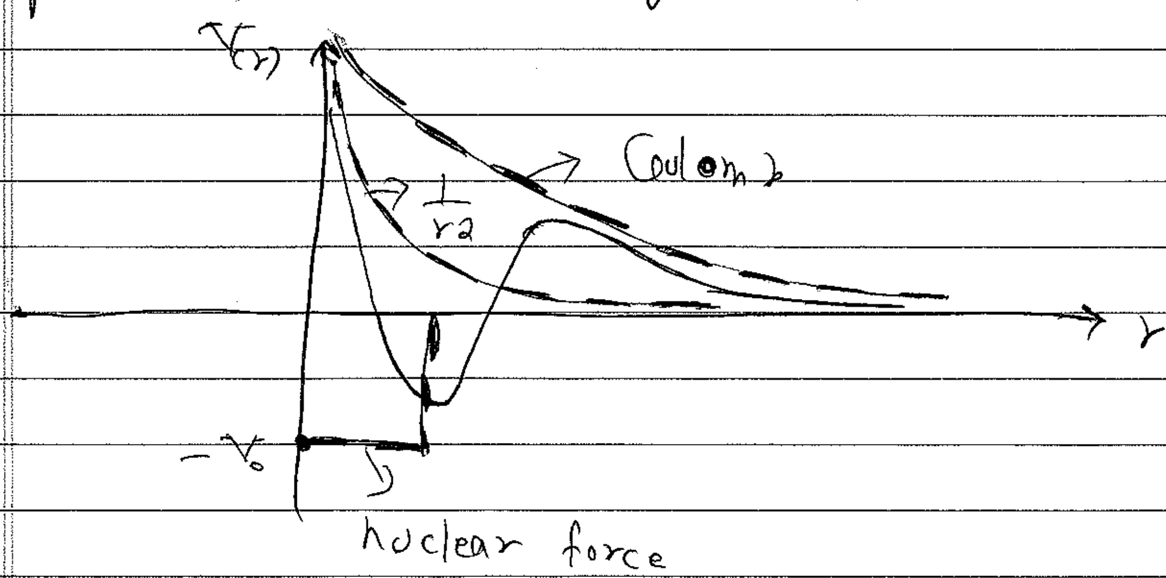
Inside a large nucleus, α particle is subject to various potentials:

1- Attractive short range potential from nuclear forces.

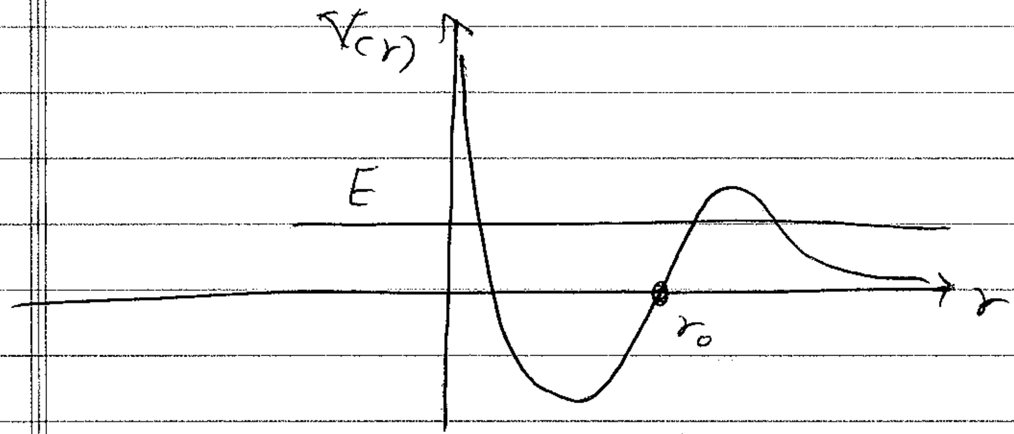
2- Repulsive Coulomb potential from other protons in the nucleus.

3- Repulsive $\frac{1}{r^2}$ potential for states with non zero angular momentum.

The combined effect leads to the following potential for the α particle:



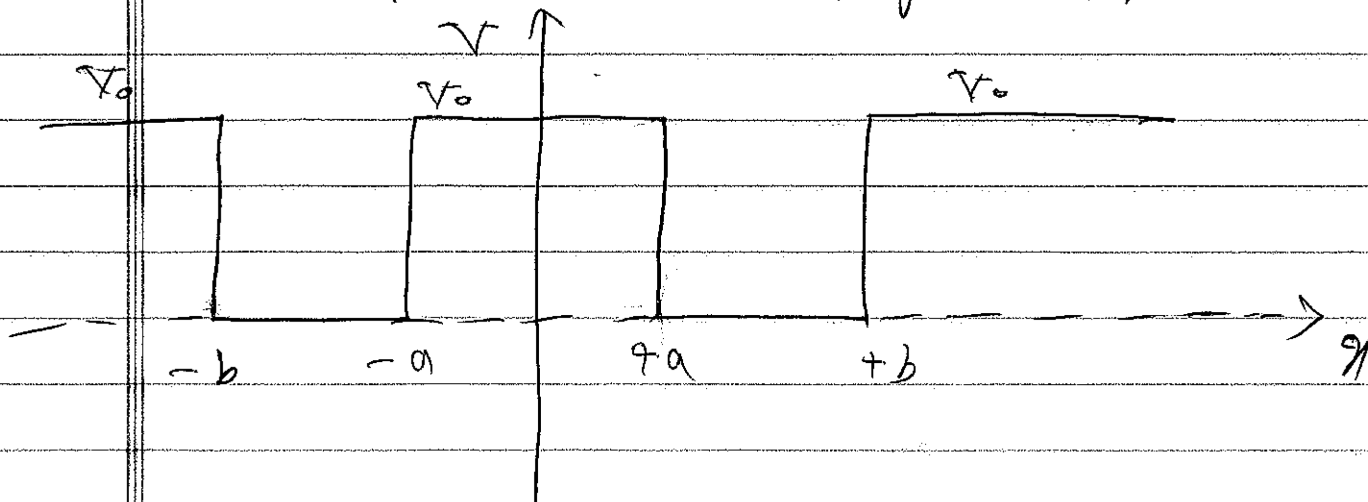
Note that for a positive energy, the eigenstate has oscillatory behavior at large r:



Therefore, an α particle localized at $r<r_0$ (contained in the nucleus) can leak out. This will result in α decay of the nucleus.

Tunneling and Removal of Degeneracy:

Consider the double well potential;



Classically, the particle exists in only one of the intervals $[-b, -a]$ or $[+a, +b]$. We therefore have a two-fold degenracy.

However, in quantum mechanics tunneling effect removes this degeneracy. The particle can be found in both wells, and the probability find it in $[-b, -a]$ interval is the same as the probability to find it in $[+a, +b]$ interval.

Instead of the classical degeneracy, we have two states with different energies: one is an even function of x (lower energy state) and one is an odd function of x (higher energy state).